International Journal of Theoretical Physics, Vol. 38, No. 11, 1999

Cosmic Strings in Realistic Particle Physics Theories and Baryogenesis

A. C. Davis[1]

Received March 9, 1999

Grand unified theories can admit cosmic strings with fermion zero modes. Such zero modes result in the string being current-carrying and the formation of stable remnants, vortons. However, the string zero modes do not automatically survive subsequent phase transitions. In this case the vortons dissipate. It is possible that the dissipating cosmic vortons create the observed baryon asymmetry of the universe. We show that fermion zero modes are an automatic consequence cosmic strings in supersymmetric theories. Since supersymmetry is not observed in nature, we consider possible supersymmetry-breaking terms. Some of these terms result in the zero modes being destroyed. We calculate the baryon asymmetry generated by the consequent dissipating cosmic vortons. If the supersymmetry-breaking scale is high enough, then the dissipating cosmic vortons could account for the observed baryon asymmetry.

1. INTRODUCTION

Many particle physics theories admit cosmic strings. For most cosmological studies the simple abelian Higgs model is used as a prototypical cosmic string theory. However, in realistic particle physics theories the situation is more complicated. The resulting cosmic strings can have a rich microstructure. Additional features can be aquired at the string core at each subsequent symmetry breaking. This additional microstructure can, in some cases, be used to constrain the underlying particle physics theory to ensure consistency with standard cosmology. For example, if the theory admits cosmic strings which aquire fermion zero modes, or bose condensates, either at formation or due to a subsequent symmetry, then the zero modes can be excited and will move up or down the string, depending on whether they are left- or

[1] Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, CB3 9EW, U.K.

0020-7748/99/11 00-2867$16.00/ 0

right-movers. This will result in the string carrying a current [1]. An initially weak current on a string loop will be amplified as the loop contracts. The current could become sufficiently strong to halt the contraction of the loop, preventing it from decaying. A stable state, or vorton [2], is formed. The density of vortons is tightly constrained by cosmological requirements. For example, if vortons are sufficiently stable so that they survive until the present time, then we require that the universe is not vorton-dominated. However, if vortons only survive a few minutes, then they can still have cosmological implications. We then require that the universe be radiation-dominated at nucleosynthesis. These requirements have been used in ref. 3 to constrain such models.

Vortons are classically stable [4], but the quantum stability is an open question. It has been assumed that, if vortons decay, they do so by quantum mechanical tunneling. This would result in them being very long-lived. However, in the case of fermion superconductivity, the existence of fermion zero modes at high energy does not guarantee that such modes survive subsequent phase transitions. The disappearance of such zero modes could give another channel for the resulting vortons to decay. Fermion zero modes could also be created at subsequent phase transitions. It is thus necessary to trace the microphysics of the cosmic string from formation through all subsequent phase transitions in the history of the universe.

For example, many popular particle physics theories above the electroweak scale are based on supersymmetry. Such theories can also admit cosmic string solutions [5]. Since supersymmetry is a natural symmetry between bosons and fermions, the fermion partner of the Higgs field forming the cosmic string is a zero mode. Thus, the particle content and interactions dictated by supersymmetry naturally give rise to current-carrying strings. Gauge symmetry breaking can arise either by introduction of a superpotential or by means of a Fayet–Iliopoulos term. In both cases fermion zero modes arise.

However, supersymmetry is not observed in nature and must therefore be broken. We consider general soft supersymmetry-breaking terms that could arise and consider the resulting effect of these on the fermion zero modes. For most soft breaking terms, the zero modes are destroyed. Hence, any vortons formed would dissipate. However, in the case of gauge symmetry breaking via a Fayet–Iliopoulos term, the zero modes, and hence vortons, survive supersymmetry breaking. Hence, supersymmetric theories which break a $U(1)$ symmetry this way would result in cosmologically stable vortons, and would therefore be ruled out. However, in the more general case, the problem of cosmic vortons seems to solve itself. That is to say, vortons will be formed at high energy, but will dissipate after the supersymmetry-breaking scale.

If the underlying supersymmetric theory is a grand unified one, then in the string core the grand unified symmetry is restored and typical grand unified processes will be unsuppressed in the string core. Once the vortons decay, the grand unified particles will be released. Their out-of-equilibrium decay results in a baryon asymmetry being produced. Depending on the scale of supersymmetry breaking, the baryon asymmetry produced could account for that required by nucleosynthesis.

In this paper, I address this problem. I first review cosmic strings in supersymmetric theories, displaying the string zero modes [5]. I then consider the effect of supersymmetry breaking on these zero modes, showing that the zero modes are destroyed in the general case [6]. The vorton density is estimated in these supersymmetric theories. I show that the underlying theory can be constrained in the case where the vortons are stable. If the vortons are unstable, I estimate resulting baryon asymmetry from dissipating cosmic vortons. I also take into account the change in entropy density from the vorton decay and show that, for supersymmetry breaking just before the vorton density dominates that of radiation, this results in a baryon asymmetry, in agreement with observation [7].

2. COSMIC STRINGS IN SUPERSYMMETRIC THEORIES

Consider supersymmetric versions of the spontaneously broken gauged $U(1)$ abelian Higgs model. These models are related to or are simple extensions of those found in ref. 8. In superfield notation, such a theory consists of a vector superfield V and m chiral superfields Φ_i ($i = 1, \ldots, m$), with $U(1)$ charges q_i. In the Wess–Zumino gauge these may be expressed in component notation as

$$V(x, \theta, \bar{\theta}) = -(\theta\sigma^{\mu}\bar{\theta})A_{\mu}(x) + i\theta^2\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}^2\theta\lambda(x) + \frac{1}{2}\theta^2\bar{\theta}^2 D(x) \quad (2.1)$$

$$\Phi_i(x, \theta, \bar{\theta}) = \phi_i(y) + \sqrt{2}\theta\psi_i(y) + \theta^2 F_i(y) \quad (2.2)$$

where $y^{\mu} = x^{\mu} + i\theta\sigma^{\mu}\bar{\theta}$. Here, ϕ_i are complex scalar fields and A_{μ} is a vector field. These correspond to the familiar bosonic fields of the abelian Higgs model. The fermions $\psi_{i\alpha}$, $\bar{\lambda}_{\alpha}$, and λ_{α} are Weyl spinors and the complex bosonic fields F_i and real bosonic field D are auxiliary fields. Finally, θ and $\bar{\theta}$ are anticommuting superspace coordinates. In the component formulation of the theory one eliminates F_i and D via their equations of motion and performs a Grassmann integration over θ and $\bar{\theta}$. Now define

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot\alpha}\,\bar\theta^{\dot\alpha}\,\partial_\mu$$

$$\bar D_{\dot\alpha} = \frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\alpha\,\sigma^\mu_{\alpha\dot\alpha}\,\partial_\mu$$

$$W_\alpha = -\frac{1}{4}\,\bar D^2\,D_\alpha\,V \tag{2.3}$$

where D_α and $\bar D_{\dot\alpha}$ are the supersymmetric covariant derivatives and W_α is the field strength chiral superfield. The superspace Lagrangian density for the theory is then given by

$$\mathcal{L} = \frac{1}{4}\,(W^\alpha W_\alpha\big|_{\theta^2} + \bar W_{\dot\alpha}\bar W^{\dot\alpha}\big|_{\bar\theta^2})$$
$$+ (\Phi_i\,e^{gq_iV}\,\Phi_i)\big|_{\theta^2\theta^2} + W(\Phi_i)\big|_{\theta^2} + \bar W(\Phi_i)\big|_{\bar\theta^2} + \kappa D \tag{2.4}$$

In this expression W is the superpotential, a holomorphic function of the chiral superfields (i.e., a function of Φ_i only and not Φ_i) and $W\big|_{\theta^2}$ indicates the θ^2 component of W. The term linear in D is known as the Fayet–Iliopoulos term [9]. Such a term can only be present in a $U(1)$ theory, since it is not invariant under more general gauge transformations.

For a renormalizable theory, the most general superpotential is

$$W(\Phi_i) = a_i\Phi_i + \frac{1}{2}\,b_{ij}\Phi_i\Phi_j + \frac{1}{3}\,c_{ijk}\Phi_i\Phi_j\Phi_k \tag{2.5}$$

with the constants b_{ij}, c_{ijk} symmetric in their indices. This can be written in component form as

$$W(\phi_i, \psi_j, F_k)\big|_{\theta^2} = a_i F_i + b_{ij}\left(F_i\phi_j - \frac{1}{2}\,\psi_i\psi_j\right) + c_{ijk}(F_i\phi_j\phi_k - \psi_i\psi_j\phi_k) \tag{2.6}$$

and the Lagrangian (2.4) can then be expanded in Wess-Zumino gauge in terms of its component fields using (2.2) and (2.1). The equations of motion for the auxiliary fields are

$$F_i^* + a_i + b_{ij}\phi_j + c_{ijk}\phi_j\phi_k = 0 \tag{2.7}$$

$$D + \kappa + \frac{g}{2}\,q_i\bar\phi_i\phi_i = 0 \tag{2.8}$$

Using these to eliminate F_i and D, we obtain the Lagrangian density in component form as

$$\mathscr{L} = \mathscr{L}_B + \mathscr{L}_F + \mathscr{L}_Y - U \tag{2.9}$$

with

$$\mathscr{L}_B = (D^{i*}_\mu \bar{\phi}_i)(D^{i\mu}\phi_i) - \frac{1}{4} F^{\mu\nu}F_{\mu\nu} \tag{2.10}$$

$$\mathscr{L}_F = -i\psi_i\sigma^\mu \, D^{i*}_\mu \bar{\psi}_i - i\lambda_i\sigma^\mu \, \partial_\mu \bar{\lambda}_i \tag{2.11}$$

$$\mathscr{L}_Y = \frac{ig}{\sqrt{2}} q_i \bar{\phi}_i \psi_i \lambda - \left(\frac{1}{2} b_{ij} + c_{ijk}\phi_k \right) \psi_i \psi_j + (\text{c.c.}) \tag{2.12}$$

$$\begin{aligned} U &= |F_i|^2 + \frac{1}{2} D^2 \\ &= |a_i + b_{ij}\phi_j + c_{ijk}\phi_j\phi_k|^2 + \frac{1}{2}\left(\kappa + \frac{g}{2} q_i \bar{\phi}_i \phi_i\right)^2 \end{aligned} \tag{2.13}$$

where $D^i_\mu = \partial_\mu + \frac{1}{2} igq_i A_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

Now consider spontaneous symmetry breaking in these theories. Each term in the superpotential must be gauge invariant. This implies that $a_i \neq 0$ only if $q_i = 0$, $b_{ij} \neq 0$ only if $q_i + q_j = 0$, and $c_{ijk} \neq 0$ only if $q_i + q_j + q_k = 0$. The situation is a little more complicated than in non-SUSY theories, since anomaly cancellation in SUSY theories implies the existence of more than one chiral superfield (and hence Higgs field). In order to break the gauge symmetry, one may either induce SSB through an appropriate choice of superpotential, or, in the case of the $U(1)$ gauge group, one may rely on a nonzero Fayet–Iliopoulos term.

I shall refer to the theory with superpotential SSB (and, for simplicity, zero Fayet–Iliopoulos term) as theory F and the theory with SSB due to a nonzero Fayet–Iliopoulos term as theory D. Since the implementation of SSB in theory F can be repeated for more general gauge groups, I expect that this theory will be more representative of general defect-forming theories than theory D, for which the mechanism of SSB is specific to the $U(1)$ gauge group.

2.1. Theory F: Vanishing Fayet–Iliopoulos Term

The simplest model with vanishing Fayet–Iliopoulos term ($\kappa = 0$) and spontaneously broken gauge symmetry contains three chiral superfields. It is not possible to construct such a model with fewer superfields which does not either leave the gauge symmetry unbroken or possess a gauge anomaly. The fields are two charged fields $\Phi_\pm$, with respective $U(1)$ charges $q_\pm = \pm 1$, and a neutral field, Φ_0. A suitable superpotential is then

$$W(\Phi_i) = \mu\Phi_0(\Phi_+\Phi_- - \eta^2) \tag{2.14}$$

with η and μ real. The potential U is minimized when $F_i = 0$ and $D = 0$. This occurs when $\phi_0 = 0$, $\phi_+\phi_- = \eta^2$, and $|\phi_+|^2 = |\phi_-|^2$. Thus we may write $\phi_\pm = \eta e^{\pm i\alpha}$, where α is some function. We shall now seek the Nielsen–Olesen [10] solution corresponding to an infinite straight cosmic string. We proceed in the same manner as for nonsupersymmetric theories. Consider only the bosonic fields (i.e., set the fermions to zero) and in cylindrical polar coordinates (r, φ, z) write

$$\phi_0 = 0 \tag{2.15}$$

$$\phi_+ = \phi_-^* = \eta e^{in\varphi} f(r) \tag{2.16}$$

$$A_\mu = -\frac{2}{g} n \frac{a(r)}{r} \delta_\mu^\varphi \tag{2.17}$$

$$F_\pm = D = 0 \tag{2.18}$$

$$F_0 = \mu\eta^2(1 - f(r)^2) \tag{2.19}$$

so that the z axis is the axis of symmetry of the defect. The profile functions $f(r)$ and $a(r)$, obey

$$f'' + \frac{f'}{r} - n^2\frac{(1-a)^2}{r^2} = \mu^2\eta^2(f^2 - 1)f \tag{2.20}$$

$$a'' - \frac{a'}{r} = -g^2\eta^2(1 - a)f^2 \tag{2.21}$$

with boundary conditions

$$f(0) = a(0) = 0$$

$$\lim_{r\to\infty} f(r) = \lim_{r\to\infty} a(r) = 1$$

Note here, in passing, an interesting aspect of topological defects in SUSY theories. The ground state of the theory is supersymmetric, but spontaneously breaks the gauge symmetry, while in the core of the defect the gauge symmetry is restored but, since $|F_i|^2 \neq 0$ in the core, SUSY is spontaneously broken there.

We have constructed a cosmic string solution in the bosonic sector of the theory. Now consider the fermionic sector. With the choice of superpotential (2.14) the component form of the Yukawa couplings becomes

$$\mathcal{L}_Y = i\frac{g}{\sqrt{2}}(\bar{\phi}_+\psi_+ - \bar{\phi}_-\psi_-)\lambda - \mu(\phi_0\psi_+\psi_- + \phi_+\psi_0\psi_- + \phi_-\psi_0\psi_+)$$
$$+ (\text{c.c.}) \tag{2.22}$$

As with a nonsupersymmetric theory, nontrivial zero-energy fermion solutions can exist around the string. Consider the fermionic ansatz

$$\psi_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi_i(r, \varphi) \tag{2.23}$$

$$\lambda = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \lambda(r, \varphi) \tag{2.24}$$

If we can find solutions for the $\psi_i(r, \varphi)$ and $\lambda(r, \varphi)$, then, following Witten, we know that solutions of the form

$$\Psi_i = \psi_i(r, \varphi) e^{\chi(z+t)}, \qquad \Lambda = \lambda(r, \varphi) e^{\chi(z+t)} \tag{2.25}$$

with χ some function, represent left-moving superconducting currents flowing along the string at the speed of light. Thus, the problem of finding the zero modes is reduced to solving for the $\psi_i(r, \varphi)$ and $\lambda(r, \varphi)$.

The fermion equations of motion derived from (2.9) are four coupled equations given by

$$e^{-i\varphi}\left(\partial_r - \frac{i}{r}\partial_\varphi\right)\bar{\lambda} - \frac{g}{\sqrt{2}}\eta f(e^{in\varphi}\psi_- - e^{-in\varphi}\psi_+) = 0 \tag{2.26}$$

$$e^{-i\varphi}\left(\partial_r - \frac{i}{r}\partial_\varphi\right)\bar{\psi}_0 + i\mu\eta f(e^{in\varphi}\psi_- + e^{-in\varphi}\psi_+) = 0 \tag{2.27}$$

$$e^{-i\varphi}\left(\partial_r - \frac{i}{r}\partial_\varphi \pm n\frac{a}{r}\right)\bar{\psi}_\pm + \eta f e^{m_p i n\varphi}\left(i\mu\psi_0 \pm \frac{g}{\sqrt{2}}\lambda\right) = 0 \tag{2.28}$$

The corresponding equations for the lower fermion components can be obtained from those for the upper components by complex conjugation, and putting $n \to -n$. The superconducting current corresponding to this solution [like (2.25), but with $\chi(t - z)$] is right-moving.

We may enumerate the zero modes using an index theorem [11], as discussed further in ref. 12. This gives $2n$ independent zero modes, where n is the winding number of the string. However, in supersymmetric theories we can calculate them explicitly using SUSY transformations. This relates the fermionic components of the superfields to the bosonic ones and we may use this to obtain the fermion solutions in terms of the background string fields. A SUSY transformation is implemented by the operator $G = e^{\xi Q + \bar{\xi}\bar{Q}}$, where ξ_α are Grassmann parameters and Q_α are the generators of the SUSY algebra, which we may represent by

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\,\partial_\mu \tag{2.29}$$

$$\bar Q^{\dot\alpha} = \frac{\partial}{\partial\bar\theta_{\dot\alpha}} - i\bar\sigma^{\mu\dot\alpha\alpha}\theta_\alpha\,\partial_\mu \tag{2.30}$$

In general such a transformation will induce a change of gauge. It is then necessary to perform an additional gauge transformation to return to the Wess–Zumino gauge in order to easily interpret the solutions. For an Abelian theory, supersymmetric gauge transformations are of the form

$$\Phi_i \to e^{-i\Lambda q_i}\Phi_i \tag{2.31}$$

$$\bar\Phi_i \to e^{i\Lambda q_i}\bar\Phi_i \tag{2.32}$$

$$V \to V + \frac{i}{g}(\Lambda - \bar\Lambda) \tag{2.33}$$

where Λ is some chiral superfield.

Consider performing an infinitesimal SUSY transformation on (2.19), using $\partial_\mu A^\mu = 0$. The appropriate Λ to return to Wess–Zumino gauge is

$$\Lambda = ig\bar\xi\bar\sigma^\mu\theta A_\mu(y) \tag{2.34}$$

The component fields then transform in the following way:

$$\phi_\pm(y) \to \phi_\pm(y) + 2i\theta\sigma^\mu\bar\xi\, D_\mu\phi_\pm(y) \tag{2.35}$$

$$\theta^2 F_0(y) \to \theta^2 F_0(y) + 2\theta\xi F_0(y) \tag{2.36}$$

$$-\theta\sigma^\mu\bar\theta A_\mu(x) \to -\theta\sigma^\mu\bar\theta A_\mu(x) + i\theta^2\bar\theta\,\frac{1}{2}\,\bar\sigma^\mu\sigma^\nu\bar\xi F_{\mu\nu}(x) - i\bar\theta^2\theta\,\frac{1}{2}\,\sigma^\mu\bar\sigma^\nu\xi F_{\mu\nu}(x) \tag{2.37}$$

Writing everything in terms of the background string fields, we find that only the fermion fields are affected to first order by the transformation. These are given by

$$\lambda_\alpha \to \frac{2na'}{gr}\,i(\sigma^z)_\alpha^\beta\xi_\beta \tag{2.38}$$

$$(\psi_\pm)_\alpha \to \sqrt{2}\left(if'\,\sigma^r m_p\,\frac{n}{r}(1-a)f\sigma^\varphi\right)_{\alpha\dot\alpha}\bar\xi^{\dot\alpha}\eta\, e^{\pm in\varphi} \tag{2.39}$$

$$(\psi_0)_\alpha \to \sqrt{2}\mu\eta^2(1-f^2)\xi_\alpha \tag{2.40}$$

where we have defined

$$\sigma^{\varphi} = \begin{pmatrix} 0 & -ie^{-i\varphi} \\ ie^{i\varphi} & 0 \end{pmatrix} \tag{2.41}$$

$$\sigma^{r} = \begin{pmatrix} 0 & e^{-i\varphi} \\ e^{i\varphi} & 0 \end{pmatrix} \tag{2.42}$$

Let us choose ξ_α so that only one component is nonzero. Taking $\xi_2 = 0$ and $\xi_1 = -i\delta/(\sqrt{2}\eta)$, where δ is a complex constant, the fermions become

$$\lambda_1 = \delta \frac{n\sqrt{2}}{g\eta}\frac{a'}{r} \tag{2.43}$$

$$(\psi_+)_1 = \delta^* \left[f' + \frac{n}{r}(1 - a)f \right] e^{i(n-1)\varphi} \tag{2.44}$$

$$(\psi_0)_1 = -i\delta\mu\eta(1 - f^2) \tag{2.45}$$

$$(\psi_-)_1 = \delta^* \left[f' - \frac{n}{r}(1 - a)f \right] e^{-i(n+1)\varphi} \tag{2.46}$$

It is these fermion solutions which are responsible for the string superconductivity. Similar expressions can be found when $\xi_1 = 0$. It is clear from these results that the string is not invariant under supersymmetry, and therefore breaks it. However, since $f'(r)$, $a'(r)$, $1 - a(r)$, and $1 - f^2(r)$ are all approximately zero outside of the string core, the SUSY breaking and the zero modes are confined to the string. We note that this method gives us two zero-mode solutions. Thus, for a winding-number-one string, we obtain the full spectrum, whereas for strings of higher winding number, only a partial spectrum is obtained.

The results presented here can be extended to non-Abelian gauge theories. This is done in ref. 6. The results are very similar to those presented here, so I leave the interested reader to consult the original paper.

2.2. Theory D: Nonvanishing Fayet–Iliopoulos Term

Now consider theory D in which there is just one primary charged chiral superfield involved in the symmetry breaking and a nonzero Fayet–Iliopoulos term. In order to avoid gauge anomalies, the model must contain other charged superfields. These are coupled to the primary superfield through terms in the superpotential such that the expectation values of the secondary chiral superfields are dynamically zero. The secondary superfields have no effect on SSB and are invariant under SUSY transformations. Therefore, for the rest of this section I shall concentrate on the primary chiral superfield which mediates the gauge symmetry breaking.

Choosing $\kappa = -\frac{1}{2}g\eta^2$, the theory is spontaneously broken and there exists a string solution obtained from the ansatz

$$\phi = \eta e^{in\varphi} f(r) \tag{2.47}$$

$$A_\mu = -\frac{2}{g} n \frac{a(r)}{r} \delta_\mu^\varphi \tag{2.48}$$

$$D = \frac{1}{2} g\eta^2(1 - f^2) \tag{2.49}$$

$$F = 0 \tag{2.50}$$

The profile functions $f(r)$ and $a(r)$ then obey the first-order equations

$$f' = n \frac{1 - a}{r} f \tag{2.51}$$

$$n \frac{a'}{r} = \frac{1}{4} g^2\eta^2(1 - f^2) \tag{2.52}$$

Now consider the fermionic sector of the theory and perform a SUSY transformation, again using Λ as the gauge function to return to Wess–Zumino gauge. To first order this gives

$$\lambda_\alpha \rightarrow \frac{1}{2} g\eta^2(1 - f^2) i(I + \sigma^z)_\alpha^\beta \xi_\beta \tag{2.53}$$

$$\psi_\alpha \rightarrow \sqrt{2} \frac{n}{r}(1 - a) f (i\sigma^r - \sigma^\varphi)_{\alpha\dot{\alpha}} \bar{\xi}^{\dot{\alpha}} \eta \, e^{in\varphi} \tag{2.54}$$

If $\xi_1 = 0$, both these expressions are zero. The same is true of all higher order terms, and so the string is invariant under the corresponding transformation. For other ξ, taking $\xi_1 = -i\delta/\eta$ gives

$$\lambda_1 = \delta g\eta(1 - f^2) \tag{2.55}$$

$$\psi_1 = 2\sqrt{2}\delta^* \frac{n}{r}(1 - a) f \, e^{i(n-1)\varphi} \tag{2.56}$$

Thus supersymmetry is only half broken inside the string. This is in contrast to theory F, which fully breaks supersymmetry in the string core. The theories also differ in that theory D's zero modes will only travel in one direction, while the zero modes of theory F (which has twice as many) travel in both directions. In both theories the zero modes and SUSY breaking are confined to the string core.

Thus, a necessary feature of cosmic strings in SUSY theories is that supersymmetry is broken in the string core and the resulting strings have fermion zero modes. As a consequence, cosmic strings arising in SUSY theories are automatically current-carrying. In general, cosmic strings arise as infinite strings or as closed loops, The usual non-current-carrying string loops decay via gravitational radiation. However, in current-carrying strings loops do not necessarily suffer the same fate. The loops could be stabilized by the angular momentum of the current carriers, forming a stable, vorton, configuration. Vortons are classically stable objects [4], though their quantum mechanical stability is an open question. The presence of vortons puts severe constraints on the underlying theory since the density of vortons could overclose the universe if vortons are stable enough to survive to the present time. If they only live for a few minutes, then the vorton density could affect nucleosynthesis. This is discussed in detail in ref. 3. However, in some theories the vorton problem solves itself.

3. SOFT SUSY BREAKING

Supersymmetry is not observed in nature. Hence, it must be broken. Supersymmetry breaking is achieved by adding soft SUSY-breaking terms which do not induce quadratic divergences.

In a general model, one may obtain soft SUSY-breaking terms by the following prescription.

1. Add arbitrary mass terms for all scalar particles to the scalar potential.
2. Add all trilinear scalar terms in the superpotential, plus their Hermitian conjugates, to the scalar potential with arbitrary coupling.
3. Add mass terms for the gauginos to the Lagrangian density.

Since the techniques we have used are strictly valid only when SUSY is exact, it is necessary to investigate the effect of these soft terms on the fermionic zero modes we have identified.

As already commented, the existence of the zero modes can be seen as a consequence of an index theorem [11]. The index is insensitive to the size and exact form of the Yukawa couplings, as long as they are regular for small r, and tend to a constant at large r. In fact, the existence of zero modes relies only on the existence of the appropriate Yukawa couplings and that they have the correct φ dependence. Thus there can only be a change in the number of zero modes if the soft breaking terms induce specific new Yukawa couplings in the theory, and it is this that we must check for. Further, it was conjectured in ref. 11 that the destruction of a zero mode occurs only when the relevant fermion mixes with another massless fermion.

I have examined each of our theories with respect to this criterion and list the results below.

3.1. Theory F

As discussed previously, the superpotential for this theory is

$$W = \mu\Phi_0(\Phi_+\Phi_- - \eta^2) \tag{3.1}$$

The trilinear and mass terms that arise from soft SUSY breaking are

$$m_0^2|\phi_0|^2 + m_-^2|\phi_-|^2 + m_+^2|\phi_+|^2 + \mu M\phi_0\phi_+\phi_- \tag{3.2}$$

The derivative of the scalar potential with respect to ϕ_0^* becomes

$$\phi_0(\mu^2|\phi_+|^2 + \mu^2|\phi_-|^2 + m_0^2) + \mu M(\phi_+\phi_-)^* \tag{3.3}$$

This will be zero at a minimum, and so $\phi_0 \neq 0$ only if $M \neq 0$.

New Higgs mass terms will alter the values of ϕ_+ and ϕ_- slightly, but will not produce any new Yukawa terms. Thus these soft SUSY-breaking terms have no effect on the existence of the zero modes.

However, the presence of the trilinear term gives ϕ_0 a nonzero expectation value, which gives a Yukawa term coupling the ψ_+ and ψ_- fields. This destroys all the zero modes in the theory since the left- and right-moving zero modes mix.

For completeness note that a gaugino mass term also mixes the left and right zero modes, aiding in their destruction.

3.2. Theory D

The $U(1)$ theory with gauge symmetry broken via a Fayet–Iliopoulos term and no superpotential is simpler to analyze. New Higgs mass terms have no effect, as in the above case, and there are no trilinear terms. Further, although the gaugino mass terms affect the form of the zero-mode solutions, they do not affect their existence, and so, in theory D, the zero modes remain even after SUSY breaking. For this class of theories, the strings remain current-carrying and hence have a potential vorton problem. This could lead to the theories being in conflict with cosmology.

4. CURRENT-CARRYING STRINGS AND VORTONS

For the theories considered in the previous sections, the strings become current-carrying due to fermion zero modes as a consequence of supersymmetry. These zero modes are present in the string core at formation. If we call the temperature of the phase transition forming the strings T_x, we can estimate the vorton density. The more general case to consider would be when the string becomes current carrying at a subsequent phase transition, but this is beyond the scope of this paper and I refer the reader to ref. 6.

The string loop is characterized by two currents, the topologically conserved phase current and the dynamically conserved particle number current. Thus the string carries two conserved quantum numbers; N is the topologically conserved integral of the phase current and Z is the particle number. A nonconducting Kibble-type string loop must ultimately decay by radiative and frictional drag processes until it disappears completely. However, a conducting string loop may be saved from disappearance by reaching a state in which the energy attains a minimum for given nonzero values of N and Z.

It should be emphasized that the existence of such vorton states does not require that the carrier field be gauge-coupled. If there is indeed a nonzero charge coupling, then the loop will have a corresponding total electric charge Q such that the particle number is $Z = Q/e$. However, the important point is that, even in the uncoupled case where Q vanishes, the particle number Z is perfectly well defined.

The physical properties of a vorton state are determined by the quantum numbers N and Z. However, these are not arbitrary. For example, to avoid decaying completely like a nonconducting loop, a conducting loop must have a nonzero value for at least one of the numbers N and Z. In fact, one would expect that both these numbers should be reasonably large compared with unity to diminish the likelihood of quantum decay by barrier tunneling. There is a further restriction on the values of their ratio Z/N in order to avoid spontaneous particle emission as a result of current saturation. In this contribution I consider the special case where $|Z| \approx N$. These are the so-called chiral vortons.

For chiral vortons we have

$$E_{\mathrm{v}} \simeq l_{\mathrm{v}} m_{\mathrm{x}}^2 \tag{4.1}$$

In order to evaluate this quantity all that remains is to work out l_{v}. Assume that vortons are approximately circular with radius given by $R_{\mathrm{v}} = l_{\mathrm{v}}/2\pi$ and angular momentum quantum number J given [13] by $J = NZ$. Thus, eliminating J, one obtains

$$l_{\mathrm{v}} \simeq (2\pi)^{1/2} |NZ|^{1/2} m_{\mathrm{x}}^{-1} \tag{4.2}$$

Thus we obtain an estimate of the vorton mass energy as

$$E_{\mathrm{v}} \simeq (2\pi)^{1/2} |NZ|^{1/2} m_{\mathrm{x}} \approx N m_{\mathrm{x}} \tag{4.3}$$

where we are assuming the classical description of the string dynamics. This is valid only if the length l_{v} is large compared with the relevant quantum wavelengths. This will only be satisfied if the product of the quantum numbers

N and Z is sufficiently large. A loop that does not satisfy this requirement will never stabilize as a vorton.

We can now calculate the vorton abundance. Assuming that the string becomes current-carrying at a scale T_x by fermion zero modes, then one expects that thermal fluctuations will give rise to a nonzero value for the topological current $|j|^2$. Hence, a random walk process will result in a spectrum of finite values for the corresponding string loop quantum numbers N and Z. Therefore, loops for which these numbers satisfy the minimum length condition will become vortons. Such loops will ultimately be able to survive as vortons if the induced current, and consequently N and Z, are sufficiently large, such that

$$|NZ|^{1/2} \gg 1 \tag{4.4}$$

Any loop that fails to satisfy this condition is doomed to lose all its energy and disappear.

The total number density of small loops with length and radial extension of the order of $L_{\min}$, the minimum length for vortons, will be not much less than the number density of all closed loops and hence

$$n \approx \nu \, L_{\min}^{-3} \tag{4.5}$$

where ν is a time-dependent parameter. The typical length scale of string loops at the transition temperature, $L_{\min}(T_x)$, is considerably greater than relevant thermal correlation length, T_x^{-1}, that characterizes the local current fluctuations. It is because of this that string loop evolution is modified after current carrier condensation. Indeed, since $L_{\min}(T_x) \gg T_x^{-1}$ and loops present at the time of the condensation satisfy $L \geq L_{\min}(T_x)$, the random walk effect can build up reasonably large and typically comparable initial values of the quantum numbers $|Z|$ and N. The expected root mean square values produced in this way from carrier field fluctuations of wavelength λ can be estimated as

$$|Z| \approx N \approx \sqrt{\frac{L}{\lambda}} \tag{4.6}$$

where $\lambda \approx T_x^{-1}$. Thus, one obtains

$$|Z| \approx N \approx \sqrt{L_{\min}(T_x) T_x} \gg 1 \tag{4.7}$$

For current condensation during the friction-dominated regime this requirement is always satisfied.

Therefore, the vorton mass density is

$$\rho_v \approx N m_x n_v \tag{4.8}$$

In the friction-dominated regime the string is interacting with the surrounding plasma. We can estimate $L_{\min}$ in this regime as the typical length scale below which the microstructure is smoothed [3]. This then gives the quantum number N,

$$N \approx \left(\frac{m_{\mathrm{P}}}{\beta T_{\mathrm{x}}}\right)^{1/4} \tag{4.9}$$

where β is a drag coefficient for the friction-dominated era that is of order unity. We then obtain the number density of mature vortons

$$n_{\mathrm{v}} \approx \nu_* \left(\frac{\beta T_{\mathrm{x}}}{m_{\mathrm{P}}}\right)^{3/2} T^3 \tag{4.10}$$

This gives the resulting mass density of the relic vorton population to be

$$\rho_{\mathrm{v}} \approx \nu_* \left(\frac{\beta T_{\mathrm{x}}}{m_{\mathrm{P}}}\right)^{5/4} T_{\mathrm{x}} T^3 \tag{4.11}$$

4.1. The Nucleosynthesis Constraint

One of the most robust predictions of the standard cosmological model is the abundances of the light elements that were fabricated during primordial nucleosynthesis at a temperature $T_{\mathrm{N}} \approx 10^{-4}$ GeV.

In order to preserve this well-established picture, it is necessary that the energy density in vortons at that time, $\rho_{\mathrm{v}}(T_{\mathrm{N}})$, should have been small compared with the background energy density in radiation, $\rho_{\mathrm{N}} \approx g^* T_{\mathrm{N}}^4$, where g^* is the effective number of degrees of freedom. Assuming that carrier condensation occurs during the friction damping regime and that g^* has dropped to a value of order unity by the time of nucleosynthesis, this gives

$$\nu_* g_s^{*\,-1} \beta^{5/4} m_{\mathrm{P}}^{-5/4} T_{\mathrm{x}}^{9/4} \ll T_{\mathrm{N}} \tag{4.12}$$

The case for which strings become current-carrying at formation has been studied previously and yields rather strong restrictions for very long-lived vortons [14]. If it is only assumed that the vortons survive for a few minutes, which is all that is needed to reach the nucleosynthesis epoch, we obtain a much weaker restriction,

$$\left(\frac{\nu_*}{g_s^*}\right)^{4/9} T_{\mathrm{x}} \ll \left(\frac{m_{\mathrm{P}}}{\beta}\right)^{5/9} T_{\mathrm{N}}^{4/9} \tag{4.13}$$

Taking $g_s^* \approx 10^2$ yields the inequality

$$T_{\mathrm{x}} \leq (\nu_*)^{-4/9} \beta^{-5/9} \times 10^9 \text{ GeV} \tag{4.14}$$

This is the condition that must be satisfied by the formation temperature of *cosmic strings that become superconducting immediately*, subject to the rather conservative assumption that the resulting vortons last for at least a few minutes. If we assume that the net efficiency factor $(\nu_*)^{-4/9}$ and drag factor $\beta^{-5/9}$ are of order unity, this condition rules out the formation of such strings during any conceivable GUT transition, but is consistent with their formation at temperatures close to that of the electroweak symmetry-breaking transition.

4.2. The Dark Matter Constraint

Let us now consider the rather stronger constraints that can be obtained if at least a substantial fraction of the vortons are sufficiently stable to last until the present epoch. It is generally accepted that the virial equilibrium of galaxies and particularly of clusters of galaxies requires the existence of a cosmological distribution of "dark" matter. This matter must have a density considerably in excess of the baryonic matter density, $\rho_b \approx 10^{-31}$ g/cm^3. On the other hand, on the same basis, it is also generally accepted that to be consistent with the formation of structures such as galaxies it is necessary that the total amount of this "dark" matter should not greatly exceed the critical closure density, namely

$$\rho_c \approx 10^{-29} \text{ g/cm}^3 \tag{4.15}$$

As a function of temperature, the critical density scales like the entropy density, so that it is given by

$$\rho_c(T) \approx g^* m_c T^3 \tag{4.16}$$

where m_c is a constant mass factor. For comparison with the density of vortons that were formed at a scale T_x we can estimate this to be

$$g_s^* m_c \approx 10^{-26} m_P \approx 10^2 \text{ eV} \tag{4.17}$$

The general dark matter constraint is

$$\Omega_v \equiv \frac{\rho_v}{\rho_c} \leq 1 \tag{4.18}$$

In the case of vortons formed as a result of condensation during the friction damping regime the relevant estimate for the vortonic dark matter fraction is obtainable from (4.11) as

$$\Omega_v \approx \beta^{5/4} \left(\frac{\nu_* m_P}{g_s^* m_c} \right) \left(\frac{T_x}{m_P} \right)^{9/4} \tag{4.19}$$

The formula (4.19) is applicable to the case considered in earlier work

[14], in which it was supposed that vortons sufficiently stable to last until the present epoch, with the strings becoming current-carrying at formation, as in the case of supersymmetric theories. In this case one obtains

$$\beta^{5/9}\,\frac{T_x}{m_P}\left(\frac{\nu_* m_P}{g_s^* m_c}\right)^{4/9} \leq 1 \tag{4.20}$$

Substituting the estimates above, we obtain

$$T_x \leq (\nu_*)^{-4/9}\beta^{-5/9} \times 10^7 \text{ GeV} \tag{4.21}$$

This result is based on the assumption that the vortons in question are stable enough to survive until the present day. Thus, this constraint is naturally more severe than its analogue in the previous section. It is to be remarked that vortons produced in a phase transition occurring at or near the limit that has just been derived would give a significant contribution to the elusive dark matter in the universe. However, if they were produced at the electroweak scale, i.e., with $T_x \approx T_s \approx T_{EW}$, where $T_{EW} \approx 10^2$ GeV, then they would constitute such a small dark matter fraction, $\Omega_v \approx 10^{-9}$, that they would be very difficult to detect.

These constraints are very general for long-lived vortons. However, if the microphysics of the underlying theory is such that the fermion zero modes are destroyed by subsequent phase transitions, then an entirely different situation pertains. For example, in our F-type SUSY theory, the zero modes did not survive supersymmetry breaking. In this case, the current, and hence the resulting vortons, would dissipate. We turn to this case in the next section. If the zero modes do survive SUSY breaking, as in the case of our D-type theory, then the theory faces a vorton problem. It seems possible that such theories are in conflict with observation.

5. DISSIPATING COSMIC VORTONS

In general, SUSY breaking occurs at a fairly low energy, in which case a sizable random current will have built up in the string loops, resulting from string self-intersections and intercommuting. When the string self-intersects or intercommutes there is a finite probability that the Fermi levels will be excited. This produces a distortion in the Fermi levels, resulting in a current flow. As a consequence, vortons will form prior to SUSY breaking.

For strings that are formed at a temperature T_x and become superconducting at formation, the vorton number density is

$$n_v = \nu_*\left(\frac{\beta T_x}{m_P}\right)^{3/2} T^3 \tag{5.1}$$

while the vorton mass density is

$$\rho_v = \nu_* \left(\frac{\beta T_x}{m_P}\right)^{5/4} T_x T^3 \quad (5.2)$$

where ν_* and β are factors of order unity.

In the F-type theory, the zero modes do not survive SUSY breaking. As a conequence, the current decays, angular momentum is lost, and the vorton shrinks and eventually decays. As the vortons decay, grand unified particles are released from the string core. Since these GUT particles are also unstable, they also decay, but in a baryon-number-violating manner. As they decay, they create a net baryon asymmetry.

Given the number density of vortons at the SUSY-breaking transition, we can estimate the baryon asymmetry produced by vorton decay using

$$\frac{n_b}{s} = \frac{n_v}{s} \epsilon K \quad (5.3)$$

where s is the entropy density, ϵ is the baryon asymmetry produced by a GUT particle, and K is the number of GUT particles per vorton. We need to consider two cases: First, the vortons may decay before they dominate the energy density of the universe and we do not need to know the time scale for vorton decay since n_v/s is an invariant quantity. Alternatively, if the vorton energy density does dominate the energy density of the universe, we must modify the temperature evolution of the universe to allow for entropy generation.

Assuming that the universe is radiation-dominated until after the electroweak phase transition, the temperature of the universe is simply that of the standard hot big bang. We can estimate the entropy density following vorton decay using the standard result

$$s = \frac{2\pi^2}{45} g^* T^3 \quad (5.4)$$

where g^* is the effective number of degrees of freedom at the electroweak scale ($\simeq 100$). The vorton-to-entropy ratio is then

$$\frac{n_v}{s} \simeq \left(\frac{T_x}{m_P}\right)^{3/2} \frac{45}{2\pi^2 g^*} \sim 5 \times 10^{-6} \quad (5.5)$$

for $T_x \sim 10^{16}$ GeV.

The number of GUT particles per vorton is obtained from (4.3),

$$K = \left(\frac{\beta T_x}{m_P}\right)^{-1/4} \sim 10 \quad (5.6)$$

and we have

$$\frac{n_b}{s} \sim 10^{-5}\epsilon \tag{5.7}$$

Alternatively, the vorton energy density may come to dominate and we must allow for a nonstandard temperature evolution. The temperature of vorton-radiation equality, T_{veq}, is given by

$$T_{\mathrm{veq}} = \frac{\nu_*}{g^*}\left(\frac{\beta T_{\mathrm{x}}}{m_{\mathrm{P}}}\right)^{5/4} T_{\mathrm{x}} \tag{5.8}$$

If we assume that the vortons decay at some temperature T_{d} and reheat the universe to a temperature T_{rh}, we have

$$\hat{g}^* T_{\mathrm{rh}}^4 = \rho_{\mathrm{v}}(T = T_{\mathrm{d}}) = \nu_*\left(\frac{\beta T_{\mathrm{x}}}{m_{\mathrm{P}}}\right)^{5/4} T_{\mathrm{x}} T_{\mathrm{d}}^3 \tag{5.9}$$

where $\hat{g}^*$ is the number of degrees of freedom for this lower temperature. This reheating and entropy generation leads to an extra baryon dilution factor. In this case the baryon asymmetry produced by the decaying vortons is given by

$$\frac{n_b}{s} = \frac{n_{\mathrm{v}}}{s} K\epsilon \left[\frac{g^*}{\hat{g}^*}\frac{T_{\mathrm{eq}}}{T_{\mathrm{d}}}\right]^{-3/4} \tag{5.10}$$

where the entropy s is that of the standard big bang model. The universe now evolves as in the standard big bang model and n_b/s remains invariant. Using the above results, the asymmetry becomes

$$\frac{n_b}{s} = \epsilon\left(\nu^* \frac{\hat{g}^{*3}}{g^{*\prime 4}}\right)^{1/4} \beta^{5/46}\left(\frac{T_{\mathrm{d}}^{12}}{m_{\mathrm{P}}^5 T_{\mathrm{x}}^7}\right)^{1/16} \tag{5.11}$$

This form is valid if the vortons dominate the energy density of the universe before they decay; if this is not the case, the dilution factor is absent and we have

$$\frac{n_b}{s} \simeq \frac{\epsilon}{g^{*\prime}}\left(\frac{T_{\mathrm{x}}}{m_{\mathrm{P}}}\right)^{5/4} \tag{5.12}$$

as above.

The maximal asymmetry produced is if the vortons decay just before they dominate the energy density. This requires $T_{\mathrm{d}} \geq 10^6$ GeV for grand unified strings. In this case, since ϵ is of order 0.01 in many GUT theories, the mechanism can easily produce

$$\frac{n_b}{s} \simeq 10^{-10} \tag{5.13}$$

as required by nucleosynthesis.

6. DISCUSSION

In this contribution I have considered the microphysics of cosmic strings arising in physical particle physics theories. In the first part I concentrated on cosmic strings in supersymmetric theories, uncovering many novel features, including the possibility of them carrying persistent currents. I then considered the fate of current-carrying string loops, showing that they form stable vortons. I was able to use this to constrain the underlying theory. I then considered the possibility, suggested in the first part, that the vortons could dissipate and, in doing so, create the observed baryon asymmetry.

In particular I investigated the structure of cosmic string solutions to supersymmetric abelian Higgs models. For completeness I analyzed two models, differing by their method of spontaneous symmetry breaking. However, I expect theory F to be more representative of general defect-forming theories, since the SSB employed there is not specific to Abelian gauge groups.

I have shown that although SUSY remains unbroken outside the string, it is broken in the string core (in contrast to the gauge symmetry, which is restored there). In theory F supersymmetry is broken completely in the string core by a nonzero F term, while in theory D supersymmetry is partially broken by a nonzero D term. I demonstrated that, due to the particle content and couplings dictated by SUSY, the cosmic string solutions to both theories are superconducting in the Witten sense. I believe this to be quite a powerful result, that all supersymmetric Abelian cosmic strings are superconducting due to fermion zero modes. An immediate and important application of the results of the present paper is that SUSY GUTs which break to the standard model and yield Abelian cosmic strings [such as some breaking schemes of $SO(10)$] must face strong constraints from cosmology [3].

While I performed this analysis for an Abelian string, the techniques are quite general and the results for non-Abelian theories are very similar.

I also analyzed the effect of soft SUSY breaking on the existence of fermionic zero modes. The Higgs mass terms did not affect the existence of the zero modes. In the theories with F-term symmetry breaking, gaugino mass terms destroyed all zero modes which involved gauginos, and trilinear terms created extra Yukawa couplings which destroyed all the zero modes present. In the theory with D-term symmetry breaking, the zero modes were unaffected by the SUSY-breaking terms. If the remaining zero modes survive subsequent phase transitions, then stable vortons could result. Such vortons would dominate the energy density of the universe, rendering the underlying GUT cosmologically problematic.

Therefore, although SUSY breaking may alleviate the cosmological disasters faced by superconducting cosmic strings [3], there are classes of string solution for which zero modes remain even after SUSY breaking. It remains to analyze all the phase transitions undergone by specific SUSY GUT models to see whether or not fermion zero modes survive down to the present time. If the zero modes do not survive SUSY breaking, the universe could experience a period of vorton domination beforehand, and then reheat and evolve as normal afterward.

I then went on to calculate the remnant vorton density, assuming that the strings become current-carrying at formation, as is the case for the supersymmetric theories under consideration. I used this density to constrain the underlying theory for the case of a persistent current. Two separate cases were considered. If the vortons survive for only a few minutes, I demanded that the universe be radiation-dominated throughout nucleosynthesis to constrain the scale of symmetry breaking to be less than 10^9 GeV. However, if the vortons survive to the present time, then one can demand that they do not overclose the universe. In this case one obtains a much stronger constraint that the scale of symmetry breaking must be less than 10^7 GeV. This suggests that GUT theories based on D-type supersymmetric theories, which would automatically predict the existence of cosmic strings with the properties we have uncovered, are in conflict with observation.

On the constructive side, I showed that it is possible for various conceivable symmetry-breaking schemes to give rise to a remnant vorton density sufficient to make up a significant portion of the dark matter in the universe.

I also showed that vortons can decay after a subsequent phase transition and these dissipating vortons can create a baryon asymmetry. For example, the zero modes in the F-type theories do not automatically survive SUSY breaking. In this case, the decaying vortons could account for the observed baryon asymmetry, depending on the scale of supersymmetry breaking. If the SUSY-breaking scale were just above the electroweak scale, then the resulting asymmetry may well not be enough. This is due to the fact that vortons dominate the energy density of the universe long before they decay. Their decay results in a reheating of the universe and an increase in the entropy density. This reheating is unlikely to have any effect on the standard cosmology following the phase transition. If, however, the scale of SUSY breaking were such that the vortons did not dominate the energy density of the universe, then their decay could explain the observed baryon asymmetry of the universe.

ACKNOWLEDGMENTS

I wish to thank my collaborators, Robert Brandenberger, Brandon Carter, Stephen Davis, Warren Perkins, and Mark Trodden, for fruitful and enjoyable collaborations. This work was supported in part by PPARC. Finally, I wish to thank Edgard Gunzig for inviting me to such a stimulating meeting and Mady Smet for allowing us to use the wonderful venue of Peyresq.

REFERENCES

[1] E. Witten, *Nucl. Phys. B* **249**, 557 (1985).
[2] R. L. Davis and E. P. S. Shellard, *Nucl. Phys. B* **323**, 209 (1989); R. L. Davis, *Phys. Rev. D* **38**, 3722 (1988).
[3] R. Brandenberger, B. Carter, A. C. Davis, and M. Trodden, *Phys. Rev. D* **54**, 6059 (1996).
[4] B. Carter and X. Martin, *Ann. Phys.* **227**, 151 (1993); X. Martin and P. Peter, *Phys. Rev. D* **51**, 4092 (1995).
[5] S. C. Davis, A. C. Davis, and M. Trodden, *Phys. Lett. B* **405**, 257 (1997).
[6] S. C. Davis, A. C. Davis, and M. Trodden, *Phys. Rev. D* **57**, 5184 (1998).
[7] A. C. Davis and W. B. Perkins, *Phys. Lett. B* **393**, 46 (1997).
[8] P. Fayet, *Nuovo Cimento A* **31**, 626 (1976).
[9] P. Fayet and J. Iliopoulos, *Phys. Lett. B* **51B**, 461 (1974).
[10] H. Nielsen and P. Olesen, *Nucl. Phys. B* **61**, 45 (1973).
[11] S. C. Davis, A. C. Davis, and W. B. Perkins, *Phys. Lett. B* **408**, 81 (1997).
[12] S. C. Davis, *Int. J. Theor. Phys.*, this issue.
[13] B. Carter, *Phys. Lett. B* **238**, 166 (1990).
[14] B. Carter, *Ann. Acad. Sci.* **647**, 758 (1991).